# Conventional Long-Term Forecasting of Demand Peak Load


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Econometric Models (Oliveira & Hellman)

Assume a fixed relationship between economic and demographic variables, on the one hand, and load growth, on the other.

End-use Models (Gellings)

Attempt to relate the current inventory of end-use equipment in a specific geographic service area to load growth.

# The Two Flaws of Conventional Models

Both sets of models are deductively equivalent to the following constant annual percentage rate of growth trajectory:

where W(t) is the peak load at time t, $A_n$ is the final historical observation, α is the average annual percentage rate of growth estimated from a set of historical observations.

$$W(t) = A_n e^{\alpha t}$$

Thus, two flaws arise:

1. The system grows without an upper bound, no matter what α is.
2. α is estimated from the most recent 8 to 10 historical observations, and hence, the system characterized is based upon less than the full information available.

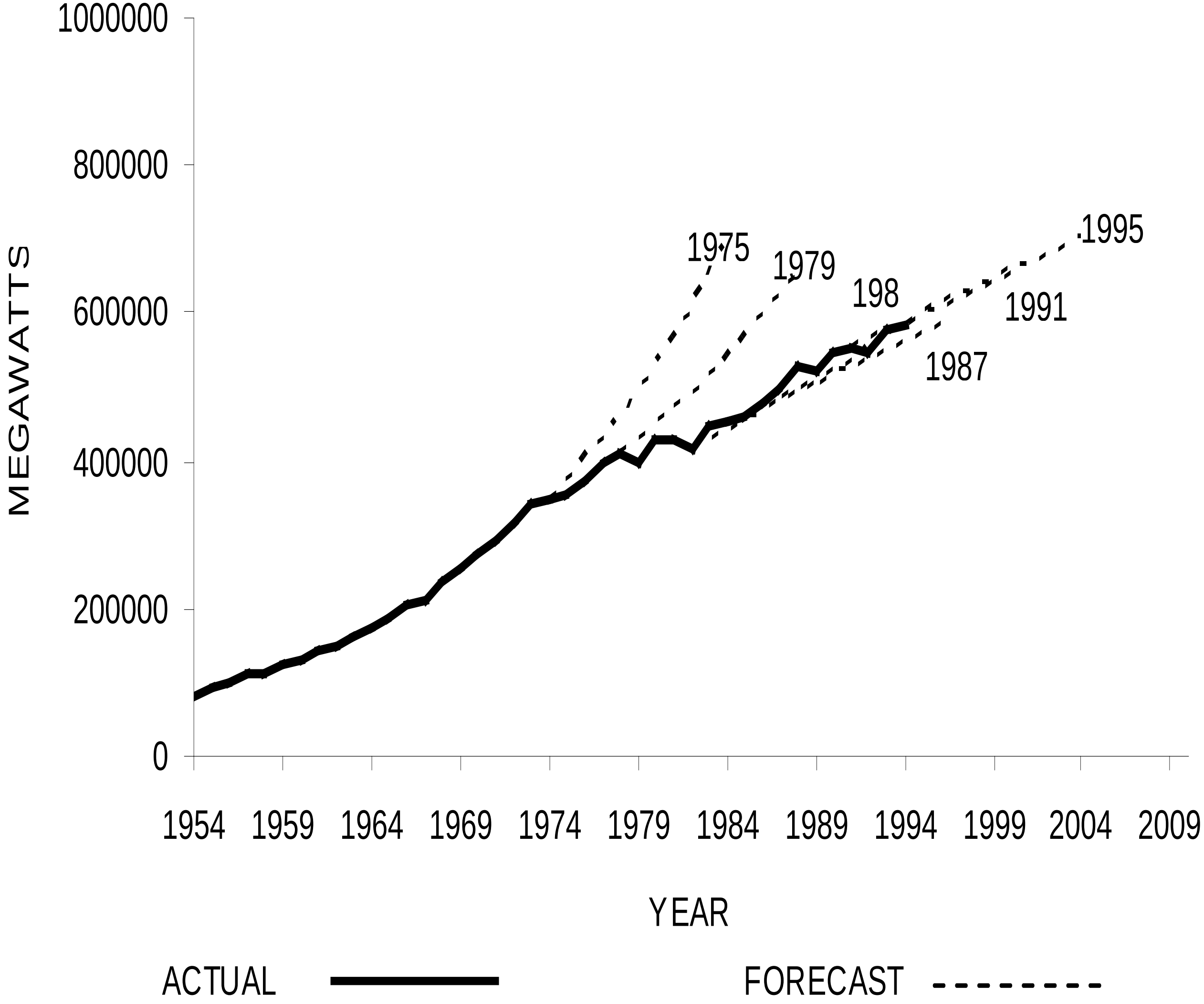


Figure 1. UNITED STATES SUMMER PEAK LOAD AS PROJECTED BY NERC

# “S” Curve Models

The question, therefore, is *what other kinds of growth models are known in science which converge to a finite upper bound, and earlier segments of the trajectory of which can be mistaken for segments from a constant rate of growth model?*

A class of growth models known as “S” curves is one answer to this question.

Such models are frequently used in biological sciences such as zoology, botany, and medicine (Von Bertalanffy).

They are also utilized in market saturation studies.

# Methodology

The domain of analysis is defined as follows:

$H_J = \langle 0, n \rangle$; is the historical interval of size n

$F_J = \langle n, n+q \rangle$; is the forecast interval of size q

$T_J = H_J \cup F_J = \langle 0, n+q \rangle$; is the domain of analysis.

The mathematical forecasting model defined over the entire domain of analysis for the time series

$W_{Ji}, i = 1,2,\ldots,n$ can be expressed as

$$W_{Jt} \equiv W_J(t) + e_{W_J(t)}, \quad t \in H_J$$

$$\left[(1-2\sigma)W_J(t) \le W_{Jt} \le (1+2\sigma)W_J(t)\right], \quad t \in T_J$$

$W_J(t)$ is the deterministic, non-negative real function, component of $W_{Jt}$ defined over $T_J$. It is assumed that $W_J(t)$ belongs to a class of “S” curve growth models.

$e_{W_J(t)} = W_{Jt} - W_J(t)$ is the stochastic component of $W_{Jt}$. It is a random variable with mean 0 and variance $\sigma^2 W_J(t)^2$; $0 \le \sigma^2 \le 0.01$.

# The Von Bertalanffy Growth Model

$$W_J(t) = \left[\alpha^{1-m} - \beta e^{-kt}\right]^{\frac{1}{1-m}}, \qquad 0 \le t < \infty$$

- $\alpha$ is the limiting growth size
- $\beta$ is a measure of the difference between the limiting growth and the initial growth
- m is the parameter that determines the proportion of the final size at which the inflection point occurs

  Monomolecular growth model (m=0)

  Logistic growth model (m=2)

  Gompertz growth model ( $m \to 1$)
- k is the rate constant of growth
- t is a measure of time

# The Generalized Von Bertalanffy Growth Model

*Analyses of historical time series such as macroeconomic magnitudes; such as population, and consumer price index, as well as many microeconomic magnitudes; such as customers sales, revenues, and peak load growth follow rather smooth "S" shaped growth patterns.*

*The smooth growth patterns, however, are altered at a limited number of significant points such as the onset and end of the Great Depression, World War II, and the OPEC price increases.*

The Von Bertalanffy model has been revised to incorporate historical and future calamities or breakthroughs that have influenced or may influence growth over time. The result is called the Generalized Von Bertalanffy model. It is a non-linear spline function of the form:

$$W_J(t) = \left[\alpha^{1-m} - \beta e^{-f(t)}\right]^{\frac{1}{1-m}}, \qquad 0 \le t < \infty$$

$$f(t) = \sum_{c=1}^{l} k_c t_c$$

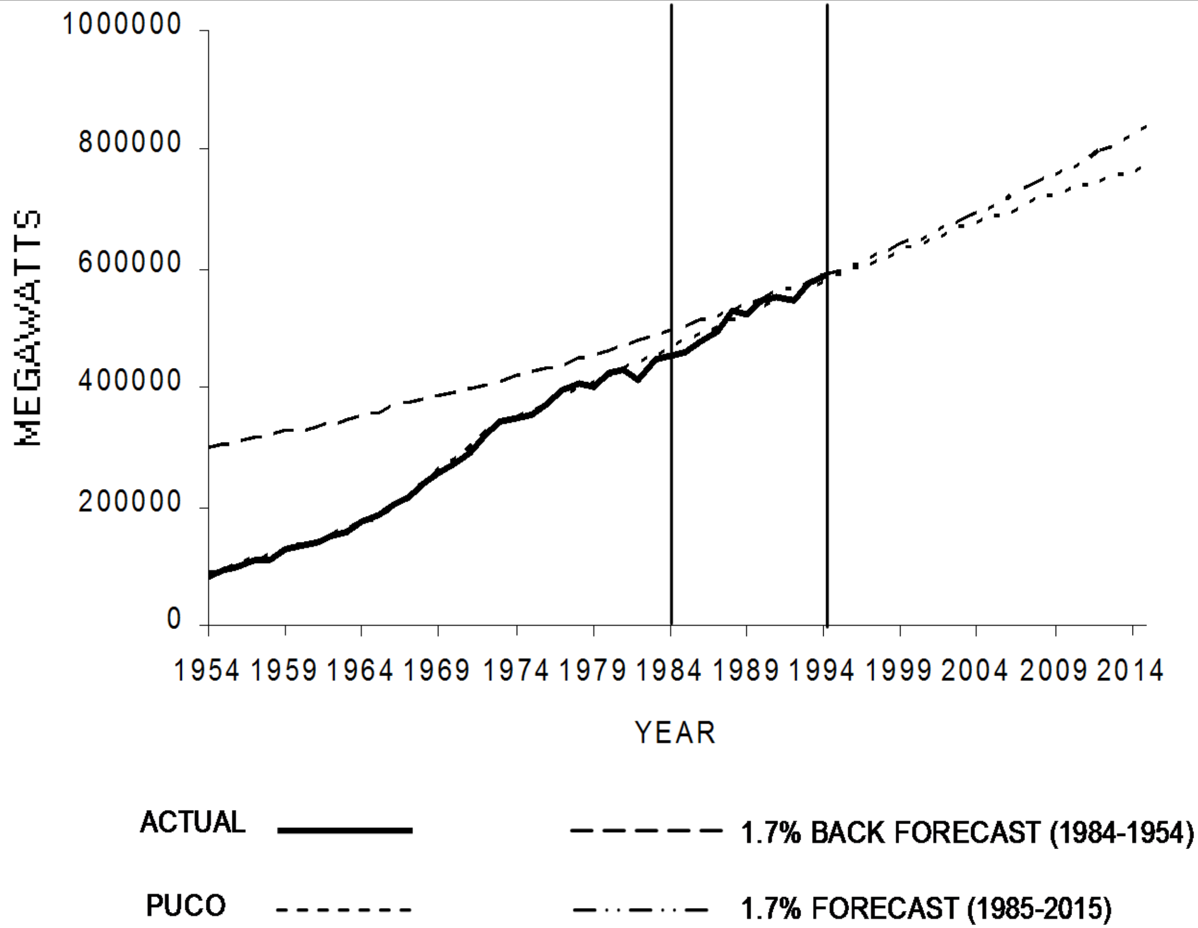


Figure 2. COMPARISON OF BACK FORECASTS AND FORECASTS PRODUCED BY FITTING A 1.7% ANNUAL RATE OF GROWTH MODEL BETWEEN 1984 AND 1994 AND PUCO FORECASTS OF U.S. SUMMER PEAK LOAD

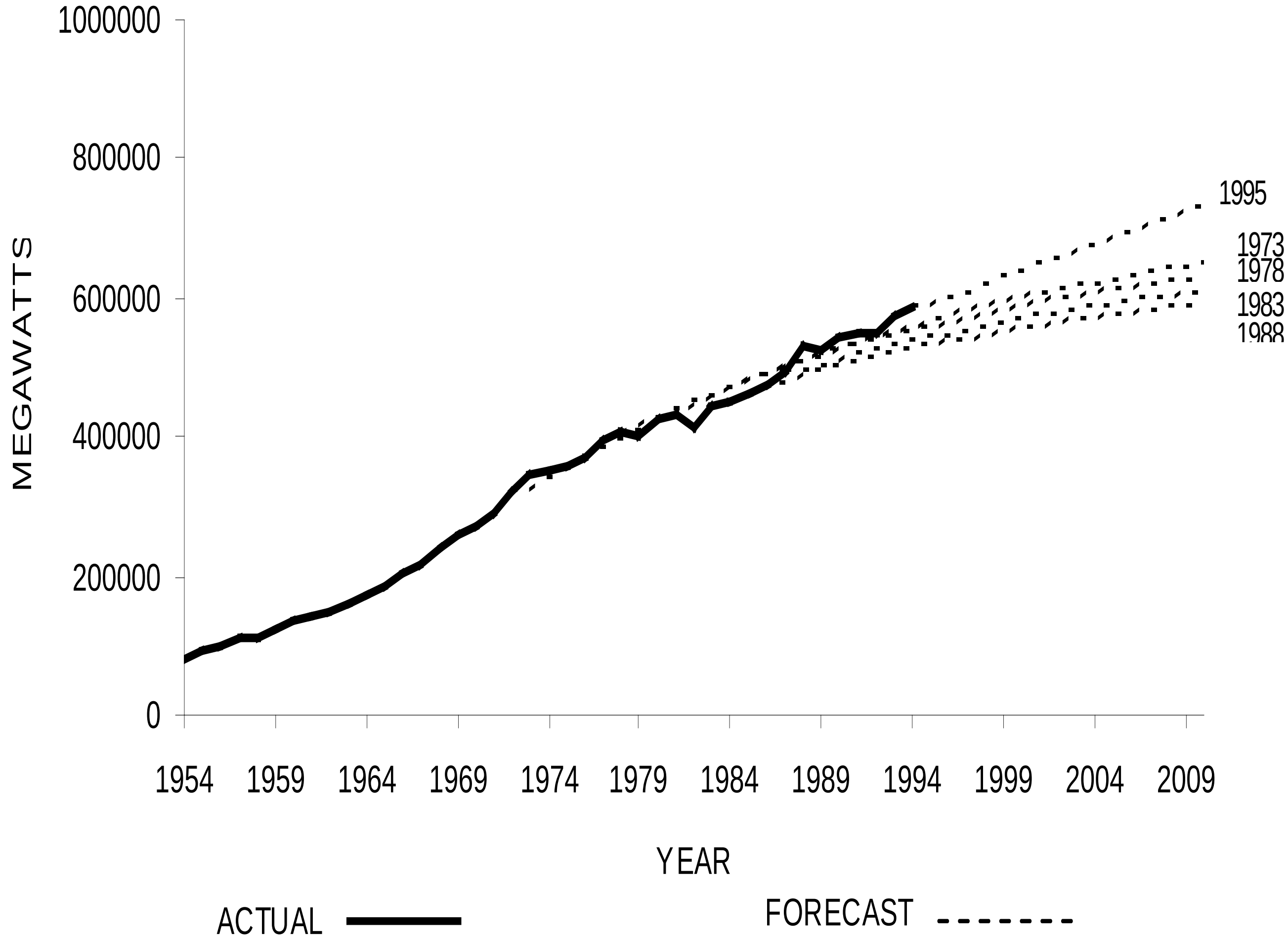


Figure 3. UNITED STATES SUMMER PEAK LOAD AS PROJECTED BY PUCO